Time-Domain Surface Impedance Boundary Conditions Enhanced by Coarse Volume Finite-Element Discretisation

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***Abstract*—The time-domain surface impedance boundary conditions allow to accurately account for the high-frequency flux components while removing the massive conducting regions from the computation domain. In this paper, a coarse volume finite-element discretization of the conductors together with a fictitious frequency-dependent conductivity are added to capture the slow varying flux components. This hybrid approach extends thus the frequency range of these impedance conditions. A 2-D test case illustrates the method.**

I. Introduction

Surface-impedance boundary conditions (SIBCs) are widely applied in frequency-domain eddy-current problems for removing the massive conducting regions from the computational domain and greatly reducing the computational cost. A necessary condition is that at the considered frequency the skin depth is sufficiently small compared to the depth or curvature of the conducting region. The higher the frequency, the better the SIBC approximation is.

Up to date, few time-domain extensions have been proposed [1], [2], [3]. In [3], the authors presented a time-domain approach based on the spatial distribution of a 1-D eddy-current problem by means of dedicated basis functions derived from the analytical frequency-domain solution. The method is developed for dual magnetodynamic formulations with both linear and nonlinear materials. However, the slow varying flux components are not properly considered.

In this paper, the frequency range of this time-domain SIBC technique is extended down to DC by introducing a coarse volume FE discretisation of the massive conducting region and a fictitious frequency-dependent conductivity. Preliminary results for a basic 1-D geometry were presented in [4]. The hybrid methodology is herein further developed and validated on a 2-D application.

II. 1-D Eddy-Current in Semi-Infinite Slab

Let us consider a magnetodynamic problem in a semi-infinite slab Ω_m ($0 \leq x \leq \infty$), with the flux density $\underline{b}(x,t)$ parallel to the z-axis. For linear and isotropic media, the conductivity σ and the permeability μ (reluctivity $\nu = 1/\mu$) are constant scalars. This 1-D problem is governed by [3]:

$$\partial_x^2 a(x,t) \;=\; \sigma\mu\,\partial_t a(x,t)\,, \tag{1}$$

with $a(x,t)$ the $y-$component of the magnetic vector potential $\underline{a}$ ($\underline{b} = \text{curl}\,\underline{a}$) and boundary condition $a(x=\infty,t) = 0$.

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Using complex notation (symbols in bold, imaginary unit $\boldsymbol{\imath} = \sqrt{-1}$), the sinusoidal steady-state solution of (1) at frequency f (skin depth $\delta = \sqrt{1/(\mu\sigma\pi f)}$) leads to the heart of the classical frequency-domain SIBC approach:

$$\partial_x \boldsymbol{a}\Big|_{x=0} \;=\; -(\delta\,\boldsymbol{Z})^{-1}\,\boldsymbol{a}(x=0)\,. \tag{2}$$

that relates the tangential components of the electric and magnetic field at the surface of the conducting region $x = 0$ via the so-called complex impedance $\boldsymbol{Z} = 1/(1+\boldsymbol{\imath})$.

A. Basis functions for the low-order time-domain model

Based on the analytical solution of (1), we choose a number of exponentially decreasing trigonometric basis functions that cover the relevant frequency range of the application to model. A set of n skin depths δ_k (frequencies f_k), $1 \leq k \leq n$, are preset for the wished accuracy and $2n$ basis functions defined [3]:

$$\alpha_{c1}(x) = e^{-x/\delta_1}\cos(x/\delta_1),$$
$$\alpha_{ck}(x) = e^{-x/\delta_k}\cos(x/\delta_k) - \alpha_{c1}(x), \quad 2 \leq k \leq n,$$
$$\alpha_{sk}(x) = e^{-x/\delta_k}\sin(x/\delta_k), \quad 1 \leq k \leq n.$$

B. Hybrid SIBC–FE approach

Let us consider a non-magnetic massive conducting 1-D region Ω_m with boundary Γ_m, depth $L = 100\,\text{mm}$ ($0 \leq x \leq L$) and $\sigma = 60\,\text{MS/m}$, which is uniformly meshed, either coarsely ($\Delta x = 5\,\text{mm}$, coarse FE solution) or finely ($\Delta x = 0.125\,\text{mm}$, reference FE solution). Supposing $\delta \ll L$ and disregarding the phase error, in Fig. 1 we compare the different approaches via the normalised impedance $Z_n = \sqrt{2}\cdot\text{abs}(\boldsymbol{Z})$ ($\boldsymbol{Z}$ in (2)). Note that Z_n equals 1 if no discretisation error is made.

The SIBC with discrete frequencies $f_1 = 1e2$, $f_2 = 1e3$ and $f_3 = 1e4$ (Hz) shows a very good accuracy in the interval $[f_1, f_3]$, i.e. Z_n close to 1, with some overshoot on both sides. The coarse FE presents a good agreement till $f_1 = 1e2$.

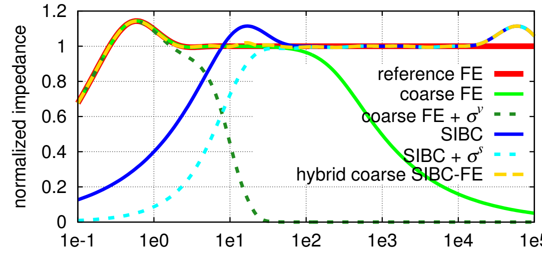

Fig. 1. Normalised impedance Z_n versus frequency

The hybrid approach consists in combining the SIBC technique with a coarse volume FE mesh of the massive conducting region for allowing slowly varying flux components.

With that purpose, we can introduce two fictitious frequency-dependent conductivities to force Z_n to drop quicker to zero on the left side of the interval $[f_1, f_3]$. We adopt σ^v for the coarse FE and σ^s for the SIBCs:

$$\sigma^v = \sigma\Big(1 + \sum_{k=1,2,\ldots} (f/c_k)^{2k}\Big) \text{ in } \Omega_m, \quad (3)$$

$$\sigma^s = \sigma\Big(1 + \sum_{k=1,2,\ldots} (c'_k/f)^{2k}\Big) \text{ on } \Gamma_m\,, \quad (4)$$

with $c_1, c_2, \ldots, c'_1, c'_2, \ldots$ properly chosen coeffients by e.g. a fitting algorithm. For the sake of simplicity, we do not consider polynomial expansions with odd powers that would amount to complex values.

The cut-off correction curves in Fig. 1 (coarse FE+σ^v, SIBC+σ^s) are obtained with the optimised values $c_1 = 12.4, c_2 = 9.1, c_3 = 9.2$ and $c'_1 = 9.4, c'_2 = 10.5, c'_3 = 9.4$. Given these two contributions in parallel at Γ_m, we get the Z_n for the hybrid approach (Fig. 1): between 0 and 1e4 Hz an excellent agreement with the reference FE solution observed. The two conductivities optimised in 1-D for a given frequency range are thus to be integrated in a higher dimensional model.

III. Integration in FE model

With our time-domain SIBC approach the volume integrals in Ω_m, appearing in the weak form of Ampère law (curl $\underline{h} = \underline{j}$), are reduced to the following surface integrals [3]:

$$(\nu\, \mathrm{curl}\, \underline{a}, \mathrm{curl}\, \underline{a}')_{\Omega_m} = \langle \underline{a}_t, \underline{a}'_t \rangle_{\Gamma_m} \cdot \nu \int_0^\infty \partial_x p\, \partial_x p'\, dx\,, \quad (5)$$

$$(\sigma\, \partial_t \underline{a}, \underline{a}')_{\Omega_m} = \langle \partial_t \underline{a}_t, \underline{a}'_t \rangle_{\Gamma_m} \cdot \sigma \int_0^\infty p\, p'\, dx\,, \quad (6)$$

with $\underline{a}_t$ the magnetic vector potential tangential to Γ_m. We take $\alpha_{ck}(x)$, $\alpha_{sk}(x)$ for the space discretisation of p and p'.

The coupling between the SIBC and the coarse volume FE method is done by imposing at Γ_m that the flux outside the massive conducting region equals the flux inside plus the SIBC component, i.e. the two fluxes are considered in parallel. In practice, this condition is directly applied to $\underline{a}$ via an additional equation at Γ_m.

When applying the hybrid SIBC–FE approach in Ω_m, the volume integral (6) reads:

$$(\sigma \partial_t \underline{a}, \underline{a}')_{\Omega_m} = (\sigma^v \partial_t \underline{a}, \underline{a}')_{\Omega_m} + \langle \partial_t \underline{a}_t, \underline{a}'_t \rangle_{\Gamma_m} \cdot \sigma^s \int_0^\infty p\, p'\, dx\,, \quad (7)$$

The fictitious conductivities $\sigma^{v,s}$, (3) and (4), can be straightforwardly expressed in the time domain by considering the relation between f^k and ∂_t^k, i.e.

$$\sigma^v \partial_t \underline{a} = \sigma\Big(\partial_t \underline{a} + \sum_{k=1,2,\ldots} (-1/2\pi c_k)^{2k} \partial_t^{2k} \underline{a}\Big) \text{ in } \Omega_m, \quad (8)$$

$$\sigma^s \partial_t \underline{a}_t = \sigma\Big(\partial_t \underline{a}_t + \sum_{k=1,2,\ldots} (-2\pi c'_k)^{2k} \partial_t^{-2k} \underline{a}_t\Big) \text{ on } \Gamma_m\,, \quad (9)$$

where ∂_t^{-2k} denotes $2k$-th integrations in time.

IV. Application example

The 2-D application example concerns a non-magnetic conducting cylinder (radius $R = 20$ cm; $\sigma = 60$ MS/m) inside an inductor. The classical FE model with a very fine discretisation of the cylinder near its surface provides an accurate reference solution. When applying the SIBC, only the mesh outside the cylinder is effectively considered. The hybrid approach requires an additional coarse volume discretisation (Fig. 2 *left*). All the approaches have been first been compared in the frequency-domain with an imposed sinusoidal current at frequency ranging from 1e-2 to 1e5 Hz. We adopt a low order approximation of the SIBC with $f_1 = 1e2$, $f_2 = 1e3$ and $f_3 = 1e4$ Hz. We adopt a low order approximation of the SIBC with $f_1 = 1e2$, $f_2 = 1e3$ and $f_3 = 1e4$ Hz. The real and imaginary part of the inductance of the source (normalised by its value at 0 Hz) is shown in Fig. 2 *right*. The hybrid approach proves accurate when the SIBC and the coarse FE are not.

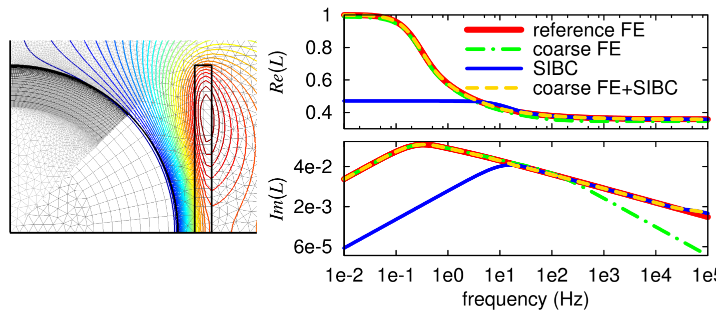

Fig. 2. Flux pattern at 100 Hz, fine and coarse FE mesh (*left*). Real and imaginary part of normalised inductance versus frequency (*right*)

Then, a sinusoidal current at 100 Hz with a DC component is imposed. A period of the flux linkage of the inductor is shown in Fig. 3 for the reference FE model, the SIBC and hybrid approaches. While the SIBC is not able to capture the DC component, the hybrid approach is. An excellent agreement with the reference FE solution is observed.

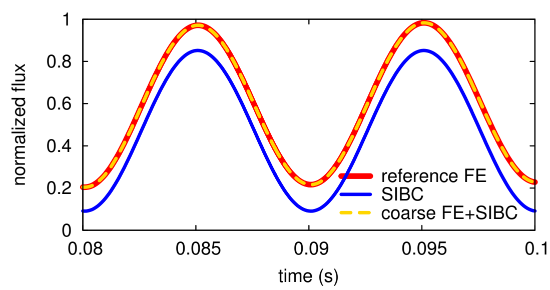

Fig. 3. Normalised magnetic flux versus time

Further results and details on the method will be given in the extended paper.

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